PAPER

Phys. Educ. **53** (2018) 035018 (6pp) iopscience.org/ped

In search of the dimensions of an incandescent light bulb filament

Luis A Ladino[1] **and Hermilda S Rondón**[2]

[1] Departmento de Ciencias Naturales, Escuela Colombiana de Ingeniería, Bogotá, Colombia
[2] Departmento de Matemáticas, Escuela Colombiana de Ingeniería, Bogotá, Colombia

E-mail: luis.ladino@escuelaing.edu.co and susana.rondon@escuelaing.edu.co

Abstract

The purpose of this paper is to present and discuss an alternative solution to an experimental problem given to high school students in the XXII Ibero-American Physics Olympiad held by Colombia this year. From the measurements of electric current and potential difference across a small tungsten filament lamp students should find the dimensions of its filament. The results obtained are compared with the ones measured directly. This challenging and low-cost experiment can be easily implemented and carried out in any introductory physics laboratory courses.

1. Introduction

This year Colombia hosted the XXII Ibero-American Physics Olympiad [1] with the participation of nineteen countries. Each country's official delegation was formed by a high school four-student team plus two academic leaders, selected on a national level. Students compete as individuals by solving challenging theoretical and experimental problems in two different sessions. The winners were awarded gold, silver or bronze medals, or an honourable mention. Among the main goals of this scholarly competition are to encourage students to study science and strengthen the bonds of friendship among the different countries in the region.

One of the experimental problems given to the students consisted of finding the length, radius and surface area of a tungsten filament of a small light bulb. The students were provided with a multimeter (students are not allowed to measure either electric current or resistance), a light bulb, a potentiometer, a fixed resistor of known resistance, a DC voltage power supply and connecting wires. Additionally, a brief overview of the theory where two topics were mainly outlined: the linear temperature dependence of resistance in a conductor and the Stefan–Boltzmann law for radiation. Along with this information, some data such as the tungsten resistivity, the temperature coefficient of resistance at room temperature and the Stefan–Boltzmann constant were given.

In this paper, we present an alternative solution to the problem proposed by the organising committee of the Olympiad by considering a more realistic dependence between resistance and temperature, namely a power law dependence [2]. We will show that the results obtained, contrary to what may be expected, are not so substantially different from those offered by the organising committee of the Olympiad. However, when the calculated value of the filament radius, using the power law relationship, is compared with the one measured directly using a light microscope the difference is smaller.

2. Problem given to students

2.1. Objective

Find the length and diameter of a small light bulb filament.

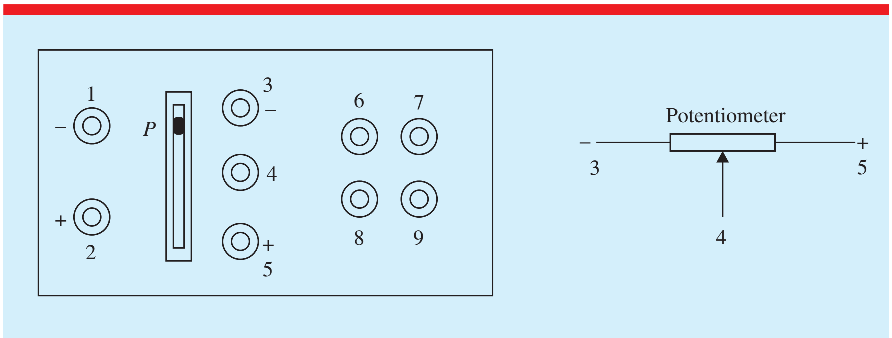

Figure 1. Box's jacks: 1–2 voltage power supply, 3–4–5 Potentiometer, 6–7 10 Ω resistor, 8–9 light bulb, P potentiometer slider knob.

2.2. Equipment

(i) A box containing a potentiometer, a 10Ω resistor and a small incandescent light bulb. Besides, it possess a set of jacks to externally connect to the elements inside it, see figure 1.
(ii) A power voltage supply with a fixed voltage.
(iii) A multimeter to measure only potential difference. It is not allowed to measure directly either electric current or resistance with the multimeter.
(iv) Several connecting wires.

2.3. Theoretical framework

The filament of an incandescent light bulb is a thin tungsten wire twisted in a spiral shape that becomes incandescent, giving off heat and light, as an electric current passes through it. The filament electric resistance varies linearly with temperature, i.e.

$$\frac{\Delta R}{\Delta T} = \alpha R_o \quad (1)$$

where R_o is the filament resistance at room temperature and $\alpha = 5.1 \times 10^{-3} K^{-1}$ is the temperature coefficient of resistance. The tungsten resistivity at 20 °C is $\rho = 5.5 \times 10^{-8}\ \Omega \cdot \mathrm{m}$.

2.4. Procedure

2.4.1. $I - V$ Characteristic curve for the light bulb.

(i) Use the provided connecting wires and the box to assemble the circuit shown in figure 2.
(ii) Measure the potential difference across the light bulb and calculate the electric current through the filament. Complete the table 1
(iii) Build the $I - V$ curve for the light bulb.

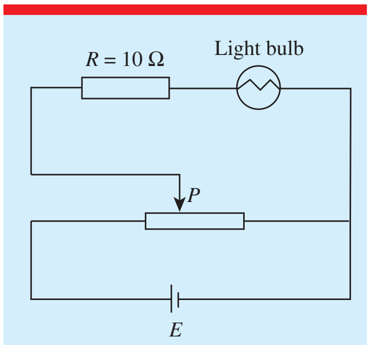

Figure 2. Schematic of the circuit to find the $i - V$ curve of the light bulb.

2.4.2. Electric resistance of the light bulb at a temperature near room temperature. It can be considered that at temperatures near room temperature the ohmic resistance of the light bulb filament follows Ohm's law and the variation of resistance with temperature is negligible.

(i) From the $I - V$ curve for the light bulb find the value for R_o at room temperature.

2.4.3. Relationship between power dissipated by the filament and its electric resistance. At high temperatures the power dissipated by the filament is mainly emitted as radiation and the rate at which this happen is given by

$$P = A\sigma T^4 \quad (2)$$

where A is the filament surface area, $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4} is the Stefan–Boltzmann constant and T its absolute temperature. It is assumed that the filament behaves as a perfect black body.

Table 1. Experimental and calculated data.

V (mV)	i (mA)	R (Ω)	T (K)	P (W)
⋮	⋮	⋮	⋮	⋮

(i) Find an expression that relates the power P and the temperature T
(ii) Plot the linearised expression found in (i).

2.4.4. Finding the surface area, length and diameter of the filament.

(i) From plot found in 2.4.3(ii) find the filament surface area.
(ii) Find the filament length.
(iii) Find the filament diameter.

2.5. Solution proposed by the organising committee of the Olympiad

Equation (2) can be written as

$$P^{\frac{1}{4}} = (\sigma A)^{\frac{1}{4}} T \tag{3}$$

and a variation in temperature is given by

$$\Delta(P^{\frac{1}{4}}) = (\sigma A)^{\frac{1}{4}} \Delta T \tag{4}$$

now, given that a linear temperature dependence of resistance is assumed, i.e. $\Delta R = \alpha R_o \Delta T$, equation (4) is rewritten as

$$\frac{\Delta(P^{\frac{1}{4}})}{\Delta R} = \frac{(\sigma A)^{\frac{1}{4}} \Delta T}{\alpha R_o \Delta T} = \frac{(\sigma A)^{\frac{1}{4}}}{\alpha R_o} = c. \tag{5}$$

Equation (5) means that the plot of $P^{\frac{1}{4}}$ against R is a straight line where c is its gradient. From equation (5)

$$A = \frac{(c\alpha R_o)^4}{\sigma}. \tag{6}$$

On the other hand, if we consider the light bulb tungsten filament shaped like a cylinder with radius r_o and length ℓ_o, its electric resistance at temperature T_o is given by $R_o = \rho \frac{\ell_o}{\pi r_o^2}$ [3], where πr_o^2 is its cross-sectional area and ρ is the tungsten resistivity. And besides the surface area of this radiator is $A = 2\pi r_0 \ell_o$. From the expression for the resistance it follows that $r_o = \sqrt{\frac{\rho \ell_o}{\pi R_o}}$; after plugging r_o into the expression for the surface area we get

$$\ell_o = \left(\frac{R_o A^2}{4\pi\rho}\right)^{\frac{1}{3}}. \tag{7}$$

The value of r_o is obtained by substituting the value of ℓ_o given by equation (7) in the expression for the surface area $A = 2\pi r_0 \ell_o$

$$r_o = \left(\frac{A\rho}{2\pi^2 R_o}\right)^{\frac{1}{3}}. \tag{8}$$

2.6. Solution proposed by the authors

Our solution differs from the previous one by assuming a power law dependence between resistance and temperature instead of a linear dependence. By taking $T \propto R^{\eta}$ in equation (2), where η is a constant to be determined, we have

$$P = Ri^2 = A\sigma T^4 = \text{const} \times R^{4\eta} \tag{9}$$

from this equation it follows that

$$i = \text{const} \times R^{\frac{4\eta - 1}{2}} \tag{10}$$

the value of η is found by linearizing the equation (10). By taking log on both sides of this equation we have that $\log i$ against $\log R$ is a stright line where m is its gradient. Therefore, the value of η is given by

$$\eta = \frac{2m+1}{4}. \tag{11}$$

Having found the value of η it follows that

$$T = \left(\frac{R}{R_0}\right)^{\eta} T_0, \tag{12}$$

where R_0 is the filament resistance at room temperature T_0. Plugging the value of T in equation (9) it follows that

$$P = A\sigma \left(\frac{T_0}{R_0^{\eta}}\right)^4 R^{4\eta}. \tag{13}$$

It is clear to see from equation (13) that there exits a linear relationship between P and $R^{4\eta}$ and the proportionality constant p is given by

$$p = A\sigma \left(\frac{T_0}{R_0^{\eta}}\right)^4. \tag{14}$$

Figure 3. Light bulb used in the experiments.

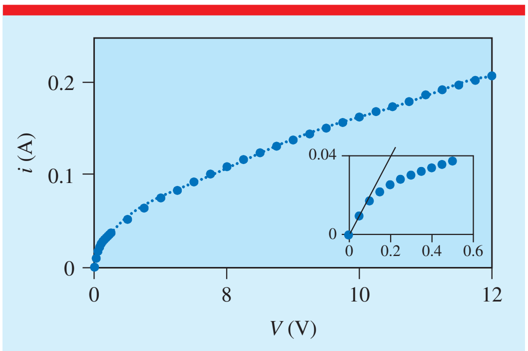

Figure 4. Plot of i against V for the light bulb filament. Inset figure shows the $I-V$ curve form low currents and the reciprocal of the straight line gradient corresponds to R_o.

Thus, by plotting P against $R^{4\eta}$ we obtain a straight line with p as its gradient, and therefore the value of effective radiating area A can be estimated as

$$A = \frac{p}{\sigma}\left(\frac{R_0^{\eta}}{T_0}\right)^4. \tag{15}$$

Following the same line of reasoning that lead to equations (7) and (8) we have $A = \frac{2\pi^2 r_o^3 R_o}{\rho}$. By equating this value of A to the one given by equation (15) we get

$$r_o = \left[\frac{p\rho}{\sigma}\frac{R_o^{4\eta-1}}{2\pi^2 T_o^4}\right]^{\frac{1}{3}}. \tag{16}$$

It is now straightforward to find ℓ_o, and its value is

$$\ell_o = \frac{A}{2\pi r_o}. \tag{17}$$

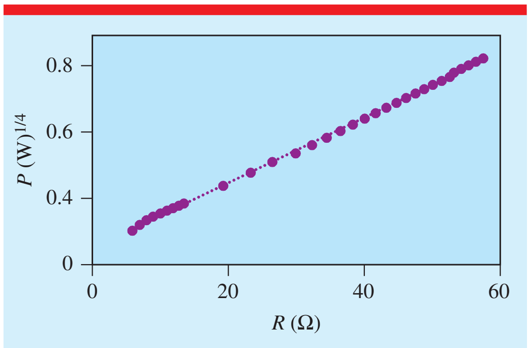

Figure 5. Plot of $P^{1/4}$ against R. Dots: experimental points. Dashed line: best fit line to the data. Pearson correlation coefficient: 0.999.

Table 2. Results obtained by assuming that $R \propto T$.

Quantity	Value
Resistance, R_o	$5.1\,\Omega$
Area, A	1.27×10^{-6} m^2
Radius, r_o	8.84×10^{-6} m
Length, ℓ_o	2.28×10^{-2} m

3. Results and discussion

Given that students were not allowed to measure either the electric current or resistance, these values must be determined indirectly. The electric current was determined as $i = \frac{V'}{R^*}$, where V' is the potential difference across the fixed resistor $R^* = 10\,\Omega$ that was connected in series to the light bulb, see figure 2. The filament resistance was determined by Ohms law, $R = \frac{V}{i}$, being V the potential difference across the light bulb. Having determined the values of R and i, it follows that the power radiated by the filament is $P = Ri^2$. In our experiment we used a 12 V, 300 mA small incandescent light bulb, see figure 3. Figure 4 shows the plot of electric current in the filament against its potential difference. From the plot, it is clear that the filament resistance certainly does not follow Ohm's law. The value of the resistance at room temperature R_o was found as the reciprocal of the line gradient at very low currents. From the inset figure, it is found that $R_o = 5.1\,\Omega$. Figure 5 shows a plot of $P^{\frac{1}{4}}$ against R. Using a linear regression on these data points, it is found that the line gradient is $c = 1.99 \times 10^{-2}\,W^{\frac{1}{4}}/\Omega$. From equations (6)–(8) the filament dimensions are found; table 2 summarizes the results.

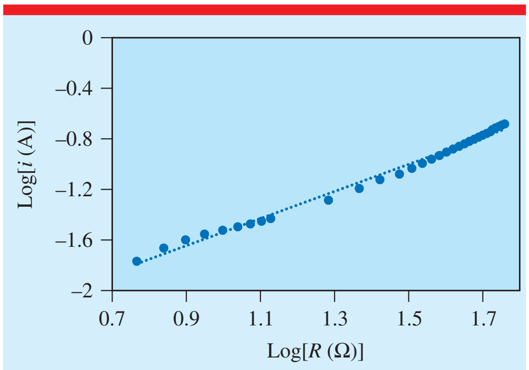

Figure 6. Plot of $\log(i)$ against $\log(R)$. Dots: experimental points. Dashed line: best fit line to the data. Pearson correlation coefficient: 0.991.

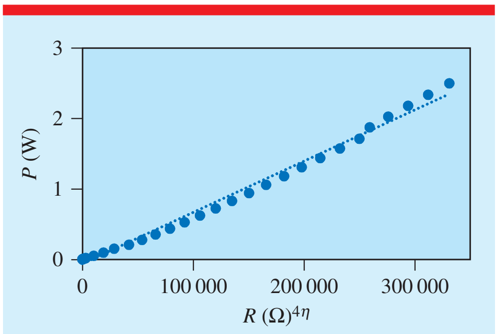

Figure 7. Plot of P against $R^{4\eta}$. Dots: experimental points. Dashed line: best fit line to the data. Pearson correlation coefficient: 0.991.

Table 3. Results obtained by assuming that $R \propto T^{\eta^{-1}}$.

Quantity	Value
Resistance, R_o	$5.1\,\Omega$
Area, A	2.89×10^{-6} m^2
Radius, r_o	1.16×10^{-5} m
Length, ℓ_o	3.95×10^{-2} m

Now, from the plot of $\log(i)$ against $\log(R)$, see figure 6, the value of the line gradient is $m = 1.07$, and from equation (11) the value of the exponent for the power law dependence between resistance and temperature exponent is given $\eta = 0.78$. Notice that the experimental data support the model reasonably well, as the square of the Pearson correlation coefficient is 0.991 and its value is fairly consistent with the value reported by Prasad and Mascarenjas [4].

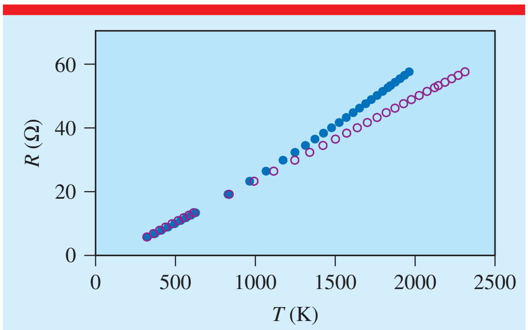

Figure 8. Plot of R against T. Filled circles, $T \propto R^{\eta}$ and open circles, $T \propto R$.

From the plot of P against $R^{4\eta}$, see figure 7, the value of the line gradient is $p = 7.30 \times 10^{-6}$, and from equations (15)–(17) the filament dimensions are found; table 3 summarizes the results.

On the other hand, the light bulb filament radius was directly measured with an ordinary light microscope and its value turned out to be $12\ \mu\text{m} \pm 1\ \mu\text{m}$. Notice that the difference between directly measured and the calculated values for the radius r_o using our model is smaller than the one obtained using the linear dependence between temperature and resistance. Finally, figure 8 shows the filament resistance as a function of its temperature using the two models. Linear dependence: $R = R_o(1 + \alpha(T - T_o))$, where $R_o = 5.1\,\Omega$, $T_o = 20 + 273.15 = 293.15\,\text{K}$ and $\alpha = 5.1 \times 10^{-3}$ K^{-1}; and the power law dependence given by equation (12). From the graphs it is clear that the light bulb filament temperatures coming from our model are lower than the ones used in the linear model.

4. Conclusions

From the measurements of the electric current and potential difference along with the two different models for the temperature dependence of the filament resistance, the dimensions of a small incandescent light bulb filament were determined. In the first model a linear dependence between resistance and temperature was assumed while in the second, a power law relation between these two quantities was taken. This latter predicts lower temperatures for the light bulb filament than the former and above all, the value for the filament radius is pretty much similar to the one measured directly with an ordinary light microscope. The results obtained confirm that the power-law temperature dependence of the resistance is a more realistic model than the linear model as is reported by Gluck and King [2] and Prasad and Mascarenjas [4]. On the other hand, it is important to highlight and commend the elegant and smart solution provided by the organising committee of the Olympiad, which offers excellent results despite its simplicity. Finally, the experiment is simple to perform and can be done with equipment easy to find in any lab and above all appropriate for either high schools students or an undergraduate physics laboratory.

Acknowledgments

The authors wish to thank lab assistants Juan C González and Francisco A Díaz for their help in the development of this experiment.

Received 13 October 2017, in final form 19 December 2017
Accepted for publication 8 January 2018
https://doi.org/10.1088/1361-6552/aaa5f0

References

[1] http://oc.uan.edu.co/9-noticias-olimpiadas/98-ganadores-olimpiada-iberoamericana-de-fisica-2017
[2] Gluck P and King J 2008 Physics of incandescent lamp burnout *Phys. Teach.* **46** 29
[3] Sanny J and Moebs W 1996 *University Physics* (Dubuque: Wm. C. Brown Publishers)
[4] Prasad B and Mascarenhas R 1978 A laboratory experiment on the application of Stefans law to tungsten lament electric lamps *Am. J. Phys.* **46** 420

Luis Alejandro Ladino Gaspar holds a BSc in physics from the Universidad Nacional, Colombia and an MSc and PhD in physics from the University of Kentucky in the US. He is a full-time physics instructor at the Escuela Colombiana de Ingeniería, Julio Garavito, Bogotá, Colombia. His primary physics interests are low-dimensional materials for thermoelectric applications, and the automatization of experiments using microcontrollers, PCs and tablets. He is also engaged in developing resources for teaching physics.

Hermilda Susana Rondón Troncoso holds a BSc in mathematics and physics from the Universidad del Tolima, Colombia, an MSc in statistics from Universidad Nacional, Colombia, and an MSc in technology in education from the Instituto Tecnológico de Monterrey, México. Since 2005, she has been working at the Departamento de Matemáticas of the Escuela Colombiana de Ingeniería, Julio Garavito, Bogotá, Colombia, and is involved in teaching probability and statistics for engineers. She is also engaged in developing resources for teaching physics.