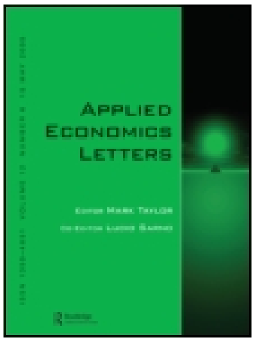

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C. Sarmiento

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Modelling firm heterogeneity with spatial 'trends'

C. SARMIENTO

Department of Agri/Business & Applied Economics, North Dakota State University, PO Box 5636, Fargo, ND 58105-5636, USA; E-mail: csarmien@ndsuext.nodak.edu

The hypothesis underlying this article is that firm heterogeneity can be captured by spatial characteristics of the firm (similar to the inclusion of a time trend in time series models). The hypothesis is examined in the context of modelling electric generation by coal powered plants in the presence of firm heterogeneity.

I. INTRODUCTION

Applied production analysis with time series data commonly uses a time trend as a proxy for technological change. This proxy is justified by the obvious correlation between time and technological change (Cooley and Prescott, 1973, Diewert and Wales, 1992).[1] As a corollary to the use of a trend variable in time series analysis, this article introduces for cross sectional analysis a proxy of firm heterogeneity based on the location of each firm. The potential usefulness of this proxy depends on the correlation between location and firm heterogeneity.

The correlation between location and firm heterogeneity stems from the existence of peculiar characteristics of firms maintained to a large extent because of transfer costs (e.g. transportation cost and institutional factors), which restrict trade, competitiveness and mobility of resources across space. Location is thus an indicator of systematic differences of firms dispersed over space, and the correlation between location and technological characteristics of the firm is likely to be stronger in production processes that involve large transfer costs.

This article constructs a proxy of firm heterogeneity based on the location of the firm, which is a two-dimensional analogue of the concept of a time trend. In an application, the proxy of heterogeneity is used to capture spatial trends generated by omitted technological differences across firms in the modelling of coal-powered technologies (a process subject to large transfer cost). The relevance of the proxy of heterogeneity is tested using a maximum likelihood (ML) estimator that incorporates spatial autoregressive residuals and instrumental variables. Statistical analysis shows significant correlation between the location of the firm and firm heterogeneity, and results show that estimates without the proxy of heterogeneity overstate the input elasticities associated with coal based electric generation. This direction of the likely bias is consistent with a case where omitted heterogeneity consists of factors of production that are positively correlated with fuel in electric generation.

II. MODELLING SPATIAL HETEROGENEITY

Assume the technology of a coal-powered plant is:

$$y_j = z(q_j, z_j; s_j) \tag{1}$$

where electric generation by firm j is y_j; quantity and quality of coal used by the firm are z_j and q_j; and fixed inputs and other variables inputs are indexed by the indicator function s_j. In terms of Jorgenson and Griliches (1967) terminology, the indicator function can be defined as all elements that contribute to coal demand but not captured in the data. For example, the indicator s_j captures technological heterogeneity (e.g. pulverizer mills of different quality)[2], and generally unreported information in electric power reports such as regional variation in the equipment and labor used at different plants.

In estimation, when firm heterogeneity s_j is omitted in Equation 1, the econometric residual is likely to show

[1] Examples of various applications that use time trends as a proxy for technology are in Griffiths *et al.* (1993), e.g. p. 491.
[2] A more refined pulverization increases combustion of coal in electric generation and, thus, increases efficiency.

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spatial correlation. Spatial correlation, in a fashion similar to temporal correlation, means that the econometric residual of a given observation is correlated with the residual of geographically nearby observations (Dubin, 1999). Spatial correlation, however, occurs in the two-dimensional space whereas temporal correlation occurs in the one-dimensional space.

An estimable form of the production function in Equation 1 is the econometric model:

$$y_j = z(q_j, z_j) + \mu_j \tag{2}$$

where μ_j is the residual and $\text{cov}(\mu_j, \mu_{j+i}) \neq 0$, for $i \neq 0$, in the presence of spatial correlation. The error term in Equation 2 can be decomposed as:

$$\mu_j = \mu_{1j} + \mu_{2j} \tag{3}$$

where μ_{1j} and μ_{2j} are random and non-random components of the spatial correlated error term and, for the non-random spatial correlation term, $E(\mu_{2j}) \neq 0$.

Kelejian and Robinson (1995) modelled the random spatial correlation term using the structure:

$$\mathbf{e}_1 = (\mathbf{I} - \rho\mathbf{W})^{-1}\boldsymbol{\mu}_1 \tag{4}$$

where $\boldsymbol{\mu}_1 = (\mu_{11}, \ldots, \mu_{1T})$ for $\mu_{1t} \sim N(0, \sigma^2)$; $\mathbf{I}$ is the identity matrix; $\mathbf{W}$ is a row standardized matrix with zeroes in the diagonal matrix and rows summing to one; and the spatial correlation coefficient is ρ.

To model non-random spatial correlation, a proxy of heterogeneity is defined in terms of a ranked representation of the geographical coordinates of the firm. In particular, assuming the longitude and latitude of firm j are L_{1j} and L_{2j}, respectively, the support of the function of firm j with the proxy of heterogeneity is:

$$\mu_{2j} = \sum_{i=1}^{2} a_i R_{ij} \tag{5}$$

where:

$$R_{ij} = \sum_{k=1}^{T} D_{ijk}$$

$$D_{ijk} = 1 \quad \text{if } (L_{ij} - L_{ik}) \geqslant 0; \text{ and}$$

$$D_{ijk} = 0 \quad \text{if } (L_{ij} - L_{ik}) < 0.$$

The rank representation in Equation 5, which resembles a spatial trend, avoids parametric nuances from modelling relative differences across firms, and guarantees sufficient variability in the proxy of heterogeneity across observations to track unobserved spatial heterogeneity. In particular, because each firm has a unique location and the technology varies over space, geographical information in Equation 5 is potentially a useful signal of technological differences across firms, and the degree of correlation between the firm's location and the underlying technology of each firm depends on the size of the transfer costs. The hypothesis underlying the article is that firm heterogeneity, s_j, can be captured by Equation 5 similar to the inclusion of a time trend in time series models.

III. APPLICATION

To estimate the production function in Equation 2, this article uses data on coal power plants. The data source contains information on the fuel usage, the quality of the fuel, electric generation, and the location of coal-powered plants. The Electric Power Annual (published by the Federal Energy Commission) reports firm level data for the year 2000 on coal usage, the coal heat content measured in BTU used by each firm, and generated megawatts. Data on location for each firm used in the sample is extracted from EPA's emissions and generation resource integrated data base (E-GRID). A total number of 268 observations are used in estimation of a log-linear form of $y(q_j, z_j)$ in Equation 2.

Ordinary least squares (OLS) estimates with and without the proxy for heterogeneity in Table 1 indicate that the

Table 1. *Input elasticity estimates of a coal powered plant production function*

Coefficient	OLS	OLS (with n.r.s.c.)	ML	ML (with n.r.s.c.)
Constant (t-stat)	9.22* (29.77)	9.12* (28.19)	9.41* (34.5)	9.34* (27.43)
Coal quantity elasticity (t-stat)	1.00* (29.44)	0.96* (28.21)	0.99* (31.38)	0.95* (29.65)
Coal quality elasticity (t-stat)	1.09* (5.56)	0.66* (2.43)	1.19* (6.92)	0.82* (3.26)
F-test of n.r.s.c.		9.45*		9.16*
F-test of r.c.			3.25*	3.64*

Notes: Models with non-random spatial correlation (n.r.s.c) include the proxy of heterogeneity. The asterisk (*) denotes statistical significance at the 5% level. The F-test of n.r.s.c. evaluates the null hypothesis of absence of non-random spatial correlation, and the F-test of r.c. evaluates the null hypothesis of absence of random spatial correlation.

proxy of heterogeneity in Equation 5 is strongly significant, and its omission is a source of bias in the probable direction indicated by Table 1. The OLS estimator may, however, be subject to endogeneity bias since in production both inputs and outputs are simultaneously determined.

An instrumental variable based on ranks

The instrumental variable (IV) estimator of the loglinear form of $y(q_j, z_j)$ in Equation 2 can be used to reduce sources of endogeneity bias. A readily available instrument for the explanatory variables in Equation 2 is the rank of the variables in the data:

$$I_j = \sum_{k=1}^{T} w_{jk} \quad \text{and} \quad H_j = \sum_{k=1}^{T} v_{jk} \tag{6}$$

where $w_{jk} = 1$ if $z_j - z_k \geqslant 0$ and $w_{jk} = 0$ if $z_j - z_k < 0$; and $v_{jk} = 1$ if $q_j - q_k \geqslant 0$ and $v_{jk} = 0$ if $q_j - q_k < 0$.[3] Haussman test in Table 2 accepts the IV estimates with the proxy of heterogeneity when using Equation 6 relative to its counterpart under the OLS estimator, and the IV estimates indicate with statistical significance the existence of spatial trends.

Modelling both random and non-random spatial correlation in a ML estimator

The OLS and IV estimators with the proxy of heterogeneity in Equation 5 capture spatial trends, but omit forms of random spatial correlation. To incorporate spatial autocorrelation terms in Equation 4, this section uses a concentrated ML estimator. The covariance matrix of the random spatial correlation term in Equation 4 is, however, specified with two weighting matrices with corresponding coefficients for spatial correlation across latitude and longitude. This generalization of Equation 4 by allowing spatial correlation to differ across latitude and longitude is relevant in coal-based technologies because coal resources categorized by quantity and quality differ more markedly from West to East than South to North. Specifically, the structure for the random spatial correlation for μ_{1t} in Equation 3 is represented by:

$$\mathbf{e}_1 = [\mathbf{I} - (\varsigma_1 \mathbf{W}_1 + \varsigma_2 \mathbf{W}_2)]^{-1} \boldsymbol{\mu}_1 \tag{7}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are row standarized matrices with spatial correlation terms ς_1 and ς_2 with respect to the longitude and latitude of the firms, respectively.

A log-linear form of Equation 2 under the structure of the residual in Equation 7 can be estimated by concentrating the likelihood function with respect to ς_1 and ς_2, which obtains the ML estimator:

$$\underset{\varsigma 1, \varsigma 2}{Max} -[\ln|\Gamma| + \ln(\mathrm{E}'\Gamma^{-1}\mathrm{E})]$$

s.t.

$$\mathbf{Q}'\Omega^{-1}\mathrm{E} = 0; \tag{8a}$$

$$\mathrm{E}'\Omega^{-1}\mathrm{E} = \mathrm{T}\sigma^2 \tag{8b}$$

where:

$$\begin{aligned}\Omega &= \sigma^2 \left\{ [\mathbf{I} - (\varsigma_1 \mathbf{W}_1 + \varsigma_2 \mathrm{W}_2)]^{-1} \left\{ [\mathbf{I} - (\varsigma_1 \mathbf{W}_1 + \varsigma_2 \mathbf{W}_2)]^{-1} \right\}' \right\}^{-1} \\ &= \sigma^2 [\mathbf{I} - (\varsigma_1 \mathbf{W}_1 + \varsigma_2 \mathbf{W}_2)]' [\mathbf{I} - (\varsigma_1 \mathbf{W}_1 + \varsigma_2 \mathbf{W}_2)]; \\ &= \sigma^2 \Gamma;\end{aligned}$$

$$\mathrm{E} = \{\ln y_1 - \Phi' \mathbf{Q}_1, \ldots, \ln y_T - \Phi' \mathbf{Q}_T\};$$

and

$$\mathbf{Q} = \{\mathbf{Q}_1, \ldots, \mathbf{Q}_T\} \quad \text{where} \quad \mathbf{Q}_i = \{1, \ln q_i, \ln z_i, R_{1i}, R_{2i}).$$

Similarly, in the context of an instrumental variable, the ML estimator of Equation 7 derived from concentrating the likelihood function with respect to ς_1 and ς_2 is

$$\underset{\varsigma 1, \varsigma 2}{Max} -[\ln|\Gamma| + \ln(\mathrm{E}'\Gamma^{-1}\mathrm{E})]$$

Table 2. *Elasticity estimates with an instrumental variable based on ranks*

Coefficient	IV	IV (with n.r.s.c.)	IV-ML	IV-ML (with n.r.s.c.)
Constant (*t*-stat)	9.18* (29.62)	8.87* (24.50)	9.56* (36.1)	8.69 (25.14)
Coal quantity elasticity (*t*-stat)	1.01* (29.74)	0.97* (28.41)	1.02* (32.88)	0.96* (26.01)
Coal quality elasticity (*t*-stat)	1.11* (5.68)	0.50 (1.82)	1.22* (7.02)	0.63* (2.61)
F-test of n.r.s.c.		9.20*		9.51*
F-test of r.c.			3.32*	3.66*
Haussman test	3.06	6.43*	3.11	7.20*

Notes: Same as the notes in Table 1. At the 5% level, the critical value of the Haussman test for using instruments for fuel quantity and quality is 5.99.

[3] Intuitively, the instrument based on ranks in Equation 6 reduces sources of feedback between the residual of the econometric model and the explanatory variable by using only qualitative information of the variable (ranking) suspicious of causing simultaneity biases. In particular, while the residual of the econometric model affects the ranking of the observations only if it also affects the variable in levels, the residual may explain the variable in levels without affecting the ranking of the variable.

s.t.

$$\underline{\mathbf{Q}}'\Omega^{-1}\mathrm{E} = 0; \tag{9a}$$

$$\mathrm{E}'\Omega^{-1}\mathrm{E} = \mathrm{T}\sigma^2 \tag{9b}$$

where $\underline{\mathbf{Q}}$ is the instrument for $\mathbf{Q}$ defined in (6).[4]

Using Gauss, coefficient estimates that maximize Equations 8 and 9 are found using a grid search over ς_1 and ς_2. Statistical significance of the spatial autocorrelation terms in Tables 1 and 2 show the relevance of the ML estimator in Equations 8 and 9 relative to the OLS and IV estimators. Input elasticity estimates derived from the concentrated likelihood function in Equations 8 and 9 are reported in Tables 1 and 2.

IV. CONCLUSION

Comparison of input elasticity estimates shows that elasticity estimates are more sensitive to spatial trends than to random spatial correlation. Specifically, elasticity estimates of the quality of coal (productivity of coal effect on output) are quite sensitive to the inclusion of spatial trends. Moreover, the spatial autocorrelation term increases the efficiency of the estimator by generating more significant *t*-statistics relative to the OLS and IV estimators in Tables 1 and 2.

Analysis of estimates shows that random spatial correlation omits information on spatial trends captured by the proxy of heterogeneity in Equation 5, while spatial trends omit information captured by the spatial autocorrelation term. Therefore, simultaneous modelling of Equations 5 and 7 is potentially useful when estimating a production function subject to large transfer costs. This result underscores an interesting spatial analogy of model specifications in time series analysis that incorporate both time trends and autocorrelation terms (Cooley and Prescott, 1973), and shows the potential need of non-random spatial correlation in future extensions to applications that include random spatial correlation in estimation (e.g. Bell and Bockstael, 2000).

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[4] The bandwidth (i.e. number of ones in the raw standardized matrix) is selected simultaneously with the spatial correlation coefficient accordingly with the criterion that maximizes the likelihood function. This criterion is valid since the same observations and parameters are defined for each bandwidth of the row standardized matrix.